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A Queueing Model with Input of MPEG Frame Sequences and Interfering Traffic

by

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Abstract

We study a queueing system having a mixture of special semi-Markov process (SSMP) and Poisson arrivals as the input process, where the Poisson arrival is regarded as interfering traffic. It is shown by numerical examples that the SSMP arrivals receive worse service than Poisson arrivals, i.e., the mean waiting time of SSMP customers is longer than that of Poisson customers.

We also propose a model of Moving Picture Experts Group (MPEG) frame arrivals as an SSMP batch arrival process. This model captures two features of the MPEG coding scheme: (i) the frequency of I, P, and B frames in a Group of Pictures (GOP), and (ii) distinct size distributions for the three frames. The waiting time of each ATM cell generated from the frames is evaluated in the numerical examples. It is found that the waiting time characteristics are rather different among some real video data.

1 Introduction

Numerous models have been proposed that characterize the feature of traffic source on a communication network. For example, Poisson (M) and interrupted Poisson processes (IPP) have been used for audio traffic, and a Markov modulated Poisson process (MMPP) [3] for video traffic. In the multimedia environment such as B-ISDN the data compression is indispensable for sending huge amount of video data. A strong candidate for such compression scheme is Moving Picture Experts Group (MPEG) [4]. Since most of the video encoding will be done using the MPEG standard, there is a need for appropriate modeling of the video traffic generated by the MPEG coding scheme.

A Transform Expand Sample (TES) and a Markov chain have been used to characterize the traffic generated with MPEG. In a TES based modeling [15], each frame type I, P and B is modeled by a TES process and these frames are interleaved in the Group of Pictures (GOP) pattern like IBBPBBPBBPBB to faithfully model an MPEG video. The queueing model with these data as an input process is simulated. On the other hand, in a Markov chain based modeling [17], Markov chains are formed for the GOP as well as scene levels by avoiding the modeling of the exact frame pattern in GOP.

The aim of this paper is to present an analytic model for evaluating the traffic characteristics of MPEG frames fed into a communication buffer together with other interfering traffic. The effects of interfering traffic have been studied by means of queues with mixed arrival processes in the past. The motivation for the queueing model with mixed arrivals is that, in the situation where many traffic sources are superposed, a tagged source is modeled closely and the other sources can be regarded as an interfering traffic all together. The GI+M/M/1 analyzed by Kuczura [7] is a queueing model having two types of arrival processes, a renewal process (GI) and a Poisson process. In [13] and [14] the service time distribution is allowed to be general (i.e. GI+M/G/1) and GI and M customers may have different service time distributions. When there is a priority between GI and M we refer to [5]. Queueing models without any waiting room are analyzed in [9] and [20]. An overview of research on the single server queues with independent GI and M input streams is provided in [14]. These queueing systems operate in continuous time. A few literatures [11], [12] deal with queueing systems that operate in discrete time. In these papers, $GI+M^{[X]}/D/1/K$ ($K<\infty$ or $K=\infty$) and $GI+M^{[X]}+B/D/1$ systems are studied respectively. Especially the departure process of GI customers is obtained in [12].

This paper consists of two parts. In the first part (Section 2), we study a queueing system having SSMP and Poisson arrivals combined as an input process, i.e. an SSMP+M/M/1 system, where the Poisson arrival is regarded as interfering traffic. The special semi-Markov process (SSMP) is a special case of the semi-Markov process such that the sojourn time in each state depends only on that state. The SSMP was introduced by Ding and Decker [1] with the aim of modeling the video traffic with variable bit rate. It can be used as the arrival process of a wide class of traffic, because it fits any marginal distribution function for interarrival times, including GI and MMPP as special cases [2]. We adapt Kuczura's approach [7] to analyze our SSMP+M/M/1 system, where the SSMP and Poisson arrivals have a common service time distribution function, namely exponential distribution. We evaluate the waiting times of both SSMP and Poisson customers, correlation of the waiting times of successive SSMP customers, as well as the jitter and interdeparture times of SSMP customers. Numerical results reveal the influence of interfering traffic on the performance of SSMP customers.

In the second part (Section 3), we propose an SSMP batch arrival process ($SSMP^{[X]}$) based on a Markov chain in which major features of the MPEG coding are incorporated. We have not modeled the scene changes and the correlation among GOPs as done in [17]. It is preferable to take these characteristics into account, but that would make the model very complicated to analyze. Hence we assume that the traffic feature is mainly affected by coding scheme, and have decided to leave the modeling at the scene and GOP levels for future work. We first analyze a generic $SSMP^{[X]}+M/M/1$ queueing system. The result is then applied to the MPEG frame sequence as the $SSMP^{[X]}$ arrival process. The Markov chain of SSMP has three states corresponding to the I, P, and B frames. The transition probabilities are determined according to the frequency of these frames in a GOP. The batch size accounts for the length in ATM cells of these frames. From the results of analysis, we can evaluate the waiting time of an arbitrary cell in the frame. Numerical examples are shown based on the data about three real video films.

2 SSMP + M/M/1

We describe the SSMP and analyze an SSMP + M/M/1 queueing system in this section. In Section 2.1 an arrival process SSMP is introduced. The queue length is analyzed in Section 2.2.

The waiting time distributions for SSMP and Poisson customers are then derived in Section 2.3. In Section 2.4 we give the joint distribution for the waiting times of successive SSMP customers. Using this result we obtain the distribution for the delay variation (or *jitter*) and the variance of interdeparture times of SSMP customers in Section 2.5. The section ends with numerical examples in Section 2.6.

2.1 SSMP

The semi-Markov process with M states is a renewal process that passes through M states at successive renewal points according to a Markov chain with transition probability matrix P [1], [2]. The sojourn time spent in state l, given that the next state is m, has distribution function $A_{lm}(x)$ and, for a given sequence of states entered, all sojourn times are independent. The SSMP is a special case of the semi-Markov process where the sojourn time in a given state depends only on that state; see Figure 1, where A_l denotes the sojourn time in state l. Hence the probability that the SSMP moves from state l to m in x time units is given by $p_{l,m}A_l(x)$, where $p_{l,m}$ is the l,m element of the transition probability matrix P of the Markov chain for the states of SSMP, and $A_l(x)$ is the sojourn time distribution in state l. We consider an SSMP with a finite number M of the states as a process governing the arrivals of customers to a queue. It is assumed that the arrivals occur at the time when the process jumps to the next state.

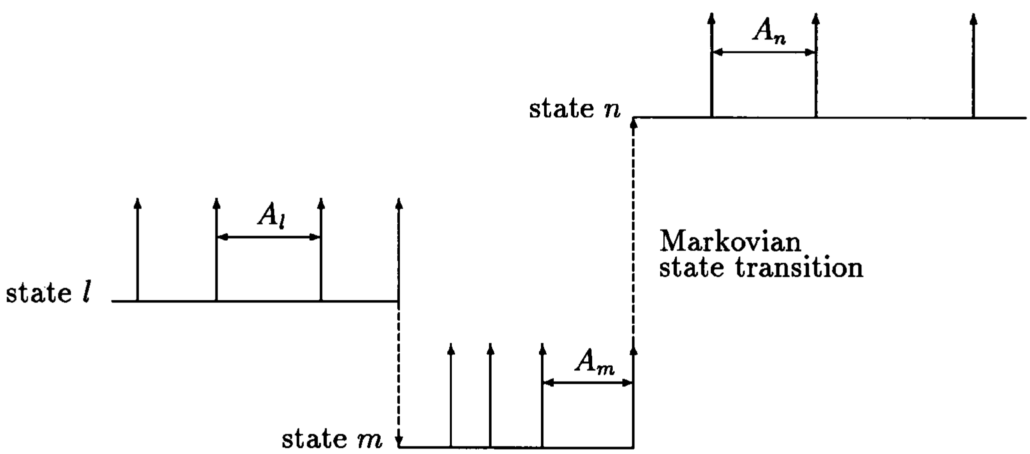

Figure 1: SSMP arrival process.

2.2 Analysis of an SSMP + M/M/1 Queueing System

We extend Kuczura's approach [7] for a GI + M/M/1 queueing system in order to analyze our SSMP + M/M/1 system. Let $Y(t)$ be the number of customers, both waiting and in service, in the system at time t. Note that between the arrival epochs of SSMP the $Y(t)$ behaves like the number of customers in an M/M/1 system. We assume that the service time distribution for both SSMP and Poisson customers is a common exponential distribution with mean $1/\mu$, and that the Poisson arrival has rate λ.

Since the number of customers present in the queue follows a birth-and-death process with birth rate λ and death rate μ between the arrival epochs of SSMP, the transition probability $P_{i,j}(t) = P\{Y(t) = j | Y(0) = i\}$ is given by [18]:

$$\begin{aligned} P_{i,j}(t) &= \rho^{\frac{1}{2}(j-i)} e^{-(\lambda+\mu)t} \Big[I_{i-j}\left(2t\sqrt{\lambda\mu}\right) + \rho^{-\frac{1}{2}} I_{i+j+1}\left(2t\sqrt{\lambda\mu}\right) \\ &+ (1-\rho) \sum_{k=1}^{\infty} \rho^{-\frac{1}{2}(k+1)} I_{i+j+k+1}\left(2t\sqrt{\lambda\mu}\right) \Big], \end{aligned} \tag{1}$$

where $\rho = \lambda/\mu$ $(0 < \rho < 1)$ and $I_j(t)$ is the modified Bessel function of the first kind of order j

$$I_j(t) = \left(\frac{t}{2}\right)^j \sum_{k=0}^{\infty} \frac{1}{k!(j+k)!} \left(\frac{t}{2}\right)^{2k}.$$

Whereas the arrival points of GI customers in a GI + M/M/1 system are regeneration points of a piecewise Markov process [8], the arrival points of SSMP customers are not regeneration points with M states in the SSMP + M/M/1 system. Therefore we study the bivariate Markovian sequence $\{(X, S)\}$ embedded at the points of SSMP arrivals, where X denotes the number of both SSMP and Poisson customers found in the system by an arriving SSMP customer, and S denotes the state of the SSMP immediately after the SSMP arrival (Figure 2). We look at the state immediately *after* the SSMP arrival. Since one-state SSMP degenerates to GI, our SSMP+M/M/1 system is an extension of the GI+M/M/1 studied in [7].

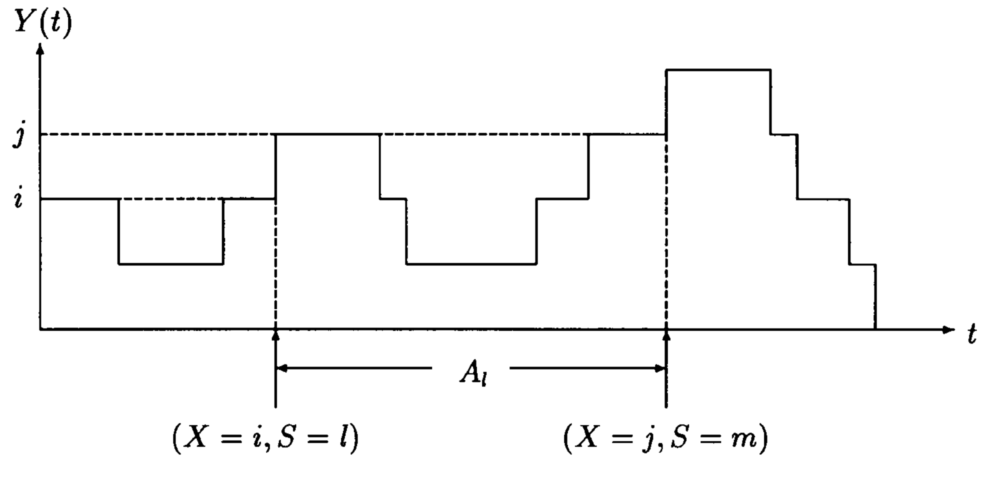

Figure 2: Number of customers and the state of SSMP.

The following notation is used in the sequel:

$A_l(t)$: interarrival time distribution in state l of the SSMP $(l = 1, 2, \dots, M)$
$r_{i,j}^{l,m}$: transition probability from state $(X = i,\ S = l)$ to state $(X = j,\ S = m)$

$p_{l,m}$: the l, m element of the transition probability matrix P $(l, m = 1, 2, \ldots, M)$
π_l : stationary probability of the matrix P $(l = 1, 2, \ldots, M)$
$P(j, m) := P(X = j, S = m)$ $(j = 0, 1, 2, \ldots; m = 1, 2, \ldots, M)$
$P(j) := P(X = j) = \sum_{m=0}^{M} P(j, m)$ $(j = 0, 1, 2, \ldots)$

where

$$r_{i,j}^{l,m} = p_{l,m} \int_0^\infty P_{i+1,j}(t) dA_l(t), \quad (i, j = 0, 1, 2, \ldots; l, m = 1, 2, \ldots, M) \tag{2}$$

and

$$\sum_{m=1}^{M} p_{l,m} = 1 \quad (l = 1, 2, \ldots, M) \quad ; \quad \sum_{l=1}^{M} \pi_l = 1$$

The probability distribution $P(j, m)$ satisfies the balance equations

$$P(j, m) = \sum_{i=0}^{\infty} \sum_{l=1}^{M} P(i, l) r_{i,j}^{l,m} = \sum_{i=0}^{\infty} \sum_{l=1}^{M} p_{l,m} P(i, l) \int_0^\infty P_{i+1,j}(t) dA_l(t) \tag{3}$$

for $j = 0, 1, 2, \ldots; m = 1, 2, \ldots, M$ with normalization condition

$$\sum_{j=0}^{\infty} \sum_{m=1}^{M} P(j, m) = 1.$$

Let $\Phi_m(z) := \sum_{j=0}^{\infty} P(j, m) z^j$. Multiplying (3) by z^j and summing over all j, we obtain

$$\Phi_m(z) = \sum_{i=0}^{\infty} \sum_{l=1}^{M} p_{l,m} P(i, l) \int_0^\infty \Gamma_{i+1}(z, t) dA_l(t), \tag{4}$$

where $\Gamma_i(z, t) := \sum_{j=0}^{\infty} P_{i,j}(t) z^j$. Whereas the function $\Gamma_i(z, t)$ is not readily available, its Laplace transform $\gamma_i(z, s) := \int_0^\infty e^{-st} \Gamma_i(z, t) dt$ has the following form [18]:

$$\gamma_i(z, s) = \frac{z^{i+1} - (1 - z)\eta^{i+1}(s)/(1 - \eta(s))}{z(s - h(z))},$$

where

$$h(z) := \frac{1}{z}(1 - z)(\mu - \lambda z)$$

and

$$\eta(s) := \frac{\lambda + \mu + s - \sqrt{(\lambda + \mu + s)^2 - 4\lambda\mu}}{2\lambda}.$$

Let us transform the real integral

$$\int_0^\infty \Gamma_{i+1}(z, t) dA_l(t)$$

appearing in (4) into a complex integral involving $\gamma_{i+1}(z, s)$ and $\alpha_l(s)$, the Laplace-Stieltjes transform (LST) of $A_l(t)$. To do so, let $dA_l(t) := e^{-at} dA_l^*(t)$, where a (> 0) is a parameter. We then have the relation

$$\int_0^\infty e^{-at} \Gamma_{i+1}(z, t) dA_l^*(t) = \frac{1}{2\pi\mathrm{i}} \int_{c-\mathrm{i}\infty}^{c+\mathrm{i}\infty} \gamma_{i+1}(z, s) \alpha_l^*(a - s) ds, \tag{5}$$

where $c > 0$, $\mathrm{i} = \sqrt{-1}$ and

$$\alpha_l^*(s) := \int_0^\infty e^{-st} dA_l^*(t) = \int_0^\infty e^{-t(s-a)} dA_l(t) = \alpha_l(s-a).$$

The integration path $\int_{c-\mathrm{i}\infty}^{c+\mathrm{i}\infty}$ is the *Bromwich integral*, and is written as $\int_{Br}$ hereafter. Substituting

$$\int_0^\infty \Gamma_{i+1}(z,t) dA_l(t) = \frac{1}{2\pi\mathrm{i}} \int_{Br} \gamma_{i+1}(z,s)\alpha_l(-s)ds \tag{6}$$

into (4), we obtain the following set of integral equations for $\Phi_m(z)$:

$$\begin{aligned}
\Phi_m(z) &= \sum_{l=1}^{M} p_{l,m} \frac{1}{2\pi\mathrm{i}} \int_{Br} \sum_{i=0}^{\infty} P(i,l)\gamma_{i+1}(z,s)\alpha_l(-s)ds \\
&= \sum_{l=1}^{M} p_{l,m} \frac{1}{2\pi\mathrm{i}z} \int_{Br} \left[\frac{z^2\Phi_l(z) - (1-z)H_l(s)}{s - h(z)}\right] \alpha_l(-s)ds, \\
&\qquad m = 1,2,\ldots,M
\end{aligned} \tag{7}$$

where

$$H_l(s) := \frac{\eta^2(s)\Phi_l(\eta(s))}{1-\eta(s)}, \qquad l = 1,2,\ldots,M. \tag{8}$$

It is assumed that $\Phi_m(z)$'s are obtained by solving (7). In Section 2.6, the explicit solution for the case $M = 2$ is shown.

We also get the probability generating function (PGF) $\Phi(z)$ for the number X of customers found in the system by an SSMP arrival, by summing $\Phi_m(z)$'s over all states

$$\Phi(z) := \sum_{j=0}^{\infty} P(j)z^j = \sum_{m=1}^{M} \Phi_m(z) = \sum_{l=1}^{M} \frac{1}{2\pi\mathrm{i}z} \int_{Br} \left[\frac{z^2\Phi_l(z) - (1-z)H_l(s)}{s-h(z)}\right] \alpha_l(-s). \tag{9}$$

2.3 Waiting Times of SSMP and Poisson Customers

We are now in a position to consider the delay distribution for an SSMP customer. Let $W(t)$ be the distribution function for the delay from the arrival instant of an SSMP customer until the beginning of his service. If $W_j(t)$ denotes the conditional delay distribution given that an arriving SSMP customer finds j other customers in the system, we have

$$W(t) = \sum_{j=0}^{\infty} W_j(t)P(j),$$

where $P(j) := P(X = j) = \sum_{m=1}^{M} P(j,m)$. Obviously, $W_0(t) = 1$ for $t \geq 0$. Assuming that the service is given in the order of arrival, we have

$$W_1(t) = 1 - e^{-\mu t},$$

and $W_j(t)$ $(j > 1)$ is the convolution of j identical distributions $W_1(t)$. If $\Omega(s)$ denotes the LST of $W(t)$, it follows that

$$\begin{aligned}
\Omega(s) &= P(0) + \sum_{j=1}^{\infty} P(j) \int_0^\infty e^{-st} dW_j(t) \\
&= P(0) + \sum_{j=1}^{\infty} P(j) \left(\frac{\mu}{s+\mu}\right)^j = \Phi\left(\frac{\mu}{s+\mu}\right).
\end{aligned} \tag{10}$$

We can then calculate the mean $E[W]$ and the second moment $E[W^2]$ of the waiting time of an SSMP customer as follows:

$$E[W] = \frac{1}{\mu} E[X], \tag{11}$$

$$E[W^2] = \frac{E[X] + E[X^2]}{\mu^2}. \tag{12}$$

where $E[X]$ and $E[X^2]$ are obtained from the PGF $\Phi(z)$ for X given in (9).

We next consider the PGF $E[z^{Y(t)}]$ for the number $Y(t)$ of customers present in the system at an arbitrary time t in the steady state. Note that this PGF is the same as that for the number of customers that the arriving Poisson customer finds according to the PASTA property. The interval between an arbitrary time and the preceding SSMP arrival time corresponds to the backward recurrence time in the Markov renewal theory. The joint backward recurrence time distribution and the probability that the SSMP is in state l is given by

$$A_l^b(y) = \lim_{t \to \infty} P(S(t) = l, V_t \leq y) = \frac{\pi_l}{E[A]} \int_0^y [1 - A_l(x)] dx, \tag{13}$$

where $S(t)$ is the state of SSMP at an arbitrary time t, V_t is the backward recurrence time at time t, and $E[A] = \sum_{l=1}^{M} \pi_l E[A_l]$ is the mean interarrival time of SSMP customers. Hence the LST $\alpha_l^b(s)$ of $A_l^b(y)$ is given by

$$\alpha_l^b(s) = \frac{\pi_l [1 - \alpha_l(s)]}{E[A] s}. \tag{14}$$

Conditioning on both the number of customers and the state at the preceding SSMP arrival point and integrating with the backward recurrence time distribution (13), the steady-state distribution of $Y(t)$ is given by

$$P(Y(t) = j) = \sum_{i=0}^{\infty} \sum_{l=1}^{M} P(i, l) \int_0^{\infty} P_{i+1,j}(y) dA_l^b(y). \tag{15}$$

Transforming (15) into the PGF and using the relation similar to (6), we get

$$\begin{aligned} E[z^{Y(t)}] &= \sum_{j=0}^{\infty} P(Y(t) = j) z^j = \sum_{i=0}^{\infty} \sum_{l=1}^{M} P(i, l) \int_0^{\infty} \Gamma_{i+1}(z, y) dA_l^b(y) \\ &= \sum_{l=1}^{M} \frac{1}{2\pi \mathrm{i}} \int_{Br} \sum_{i=0}^{\infty} P(i, l) \gamma_{i+1}(z, s) \alpha_l^b(-s) ds \\ &= \sum_{l=1}^{M} \frac{1}{2\pi \mathrm{i} z} \int_{Br} \left[\frac{z^2 \Phi_l(z) - (1 - z) H_l(s)}{s - h(z)} \right] \alpha_l^b(-s) ds, \end{aligned} \tag{16}$$

where $H_l(s)$ and $\alpha_l^b(s)$ are given in (8) and (14), respectively. The LST of the distribution function for the waiting time of a Poisson customer is then given by setting $z = \mu/(s + \mu)$ in (16).

2.4 Joint LST of the Waiting Times of Successive SSMP Customers

We proceed to derive the joint PGF $\Phi(z_1, z_2)$ of the numbers X_n and X_{n+1} of customers that the nth and the $n+1$st SSMP customers find at their arrival times.

Let $\Phi_{l,m}(z_1, z_2)$ be the joint PGF of X_n and X_{n+1} and the probability that $S_n = l$ and $S_{n+1} = m$. The joint distribution of (X_n, S_n) and (X_{n+1}, S_{n+1}) is given by

$$\begin{aligned}
&P(X_{n+1} = j, S_{n+1} = m, X_n = i, S_n = l) \\
&= P(X_{n+1} = j, S_{n+1} = m | X_n = i, S_n = l)\ P(X_n = i, S_n = l) \\
&= P(X_n = i, S_n = l)\ r_{i,j}^{l,m} \\
&= P(X_n = i, S_n = l)\ p_{l,m} \int_0^\infty P_{i+1,j}(t) dA_l(t).
\end{aligned}$$

Transforming into the PGF with respect to X_{n+1}, we get

$$\begin{aligned}
\Phi_{l,m}(X_n = i, z_2) &:= \sum_{j=0}^{\infty} P(X_{n+1} = j, S_{n+1} = m, X_n = i, S_n = l) z_2^j \\
&= P(X_n = i, S_n = l)\ p_{l,m} \int_0^\infty \Gamma_{i+1}(z_2, t) dA_l(t) \\
&= P(i, l)\ p_{l,m} \frac{1}{2\pi \mathrm{i}} \int_{Br} \gamma_{i+1}(z_2, s) \alpha_l(-s) ds.
\end{aligned}$$

where we have used (6). Further transforming into the PGF with respect to X_n, we obtain

$$\begin{aligned}
\Phi_{l,m}(z_1, z_2) &= \sum_{i=0}^{\infty} \Phi_{l,m}(X_n = i, z_2) z_1^i = p_{l,m} \frac{1}{2\pi \mathrm{i}} \int_{Br} \sum_{i=0}^{\infty} P(i, l) \gamma_{i+1}(z_2, s) z_1^i \alpha_l(-s) ds \\
&= p_{l,m} \frac{1}{2\pi \mathrm{i} z_2} \int_{Br} \frac{z_2^2(1 - \eta(s)) \Phi_l(z_1 z_2) - (1 - z_2) \eta^2(s) \Phi_l(\eta(s) z_1)}{(s - h(z_2))(1 - \eta(s))} \alpha_l(-s) ds.
\end{aligned} \tag{17}$$

Thus we get $\Phi(z_1, z_2)$ by summing $\Phi_{l,m}(z_1, z_2)$ over l and m as

$$\begin{aligned}
\Phi(z_1, z_2) &= \sum_{l=1}^{M} \sum_{m=1}^{M} \Phi_{l,m}(z_1, z_2) \\
&= \sum_{l=1}^{M} \frac{1}{2\pi \mathrm{i} z_2} \int_{Br} \frac{z_2^2(1 - \eta(s)) \Phi_l(z_1 z_2) - (1 - z_2) \eta^2(s) \Phi_l(\eta(s) z_1)}{(s - h(z_2))(1 - \eta(s))} \alpha_l(-s) ds.
\end{aligned} \tag{18}$$

Note that the joint PGF $\Phi(z_1, z_2)$ is expressed in terms of $\Phi_l(z)$ given in (7). Also note that $\Phi(1, z) = \Phi(z)$ in (9) as the marginal PGF.

The joint LST $\Omega(s_1, s_2)$ for the waiting times of the nth and the $n+1$st SSMP customers can be obtained by conditioning on the number of customers at the arrival times. Thus we get

$$\begin{aligned}
\Omega(s_1, s_2) &= \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \left(\frac{\mu}{s_1 + \mu} \right)^i \left(\frac{\mu}{s_2 + \mu} \right)^j P(X_n = i, X_{n+1} = j) \\
&= \Phi \left(\frac{\mu}{s_1 + \mu}, \frac{\mu}{s_2 + \mu} \right).
\end{aligned} \tag{19}$$

2.5 Jitter and Departure Process

In this section the delay variation (jitter [10]) is studied as a measure of distortion of the arrival process through the server. The departure process of SSMP customers is also studied and the variance of their interdeparture times is obtained.

Let Q_n denote the queueing delay of the nth SSMP customer, including the service time. The delay jitter of the nth customer J_n is defined as

$$J_n = Q_{n+1} - Q_n \qquad n \geq 1, \tag{20}$$

namely the difference in the queueing times of the nth and the $n+1$st customers.

If A_n and D_n are the interarrival and the interdeparture time of the nth and the $n+1$st SSMP customers, respectively, an equivalent definition of the jitter is given by

$$J_n = D_n - A_n \qquad n \geq 1, \tag{21}$$

because

$$\begin{aligned} D_n &= d_{n+1} - d_n \\ &= t_{n+1} - t_n + Q_{n+1} - Q_n \\ &= A_n + Q_{n+1} - Q_n, \end{aligned} \tag{22}$$

where t_n and d_n are the arrival and the departure times of the nth SSMP customer, respectively, and $d_n = t_n + Q_n$.

The jitter can take both positive and negative values. Taking a value close to 0 means that the degree of the distortion of the arrival process is small in the sense that the difference of interarrival and interdeparture times is small. Negative jitter means that the interval between customers becomes smaller by passing through the queue. This phenomenon is called *clumping* or *clustering* [10]. On the other hand, positive jitter means that the interval between customers becomes bigger by passing through the queue.

We use the result (18) and (19) to obtain the LST $E[e^{-sJ}]$ of the jitter J in the steady state. Since equation (19) is for the successive waiting times, we obtain the joint LST of the successive queueing times Q_n and Q_{n+1} by adding the independent service times. Hence the LST $E[e^{-sJ}]$ is given by

$$\begin{aligned} E[e^{-sJ}] &= E[e^{-s(Q_{n+1}-Q_n)}] = E[e^{-s_1Q_n}e^{-s_2Q_{n+1}}]\Big|_{s_1=-s, s_2=s} \\ &= \frac{\mu}{s_1+\mu}\frac{\mu}{s_2+\mu}\Phi\left(\frac{\mu}{s_1+\mu}, \frac{\mu}{s_2+\mu}\right)\Bigg|_{s_1=-s, s_2=s} \\ &= \frac{\mu^2}{\mu^2-s^2}\Phi\left(\frac{\mu}{\mu-s}, \frac{\mu}{\mu+s}\right). \end{aligned} \tag{23}$$

In the steady state the mean of jitter $E[J]$ equals 0 as seen from the definition. The variance of jitter is given by differentiating (23) twice with respect to s and then setting $s = 0$.

$$Var[J] = \frac{d^2}{ds^2}E[e^{-sJ}]\Big|_{s=0} = \frac{2}{\mu^2} + \frac{d^2}{ds^2}\Phi\left(\frac{\mu}{\mu-s}, \frac{\mu}{\mu+s}\right)\Bigg|_{s=0}. \tag{24}$$

We next consider the departure process of SSMP customers. Since it seems difficult to obtain the LST of the interdeparture interval, we calculate the mean $E[D]$ $(= E[A])$ and the second moment of the interdeparture interval D. Forming the expectation of both sides of (22) and then taking the limit $n \to \infty$ yields

$$E[D] = E[A]. \tag{25}$$

Squaring (22) and then taking expectation we obtain

$$\begin{aligned} E[D^2] &= E[(A_n + Q_{n+1} - Q_n)^2] \\ &= E[A^2] + 2E[Q^2] - 2E[Q]E[A] - 2E[Q_n Q_{n+1}] + 2E[A_n Q_{n+1}]. \end{aligned} \tag{26}$$

Note that though the queueing time Q_n of the nth SSMP customer and the interarrival time A_n of the nth and the $n+1$st SSMP customers are independent, Q_{n+1} and A_n are *not* independent. $E[A]$ and $E[A^2]$ are given quantities, and $E[Q]$ and $E[Q^2]$ can be obtained from (10). $E[Q_n Q_{n+1}]$ is obtained from (19). Thus the only unknown in (26) is $E[A_n Q_{n+1}]$, which we will calculate from the joint LST $E[e^{-s_1 Q_{n+1}} e^{-s_2 A_n}]$ in the following.

By conditioning on the state immediately after the arrival of the nth SSMP customer, we have

$$E[e^{-s_1 Q_{n+1}} e^{-s_2 A_n}] = \sum_{l=1}^{M} E[e^{-s_1 Q_{n+1}} e^{-s_2 A_l} | S_n = l] P(S_n = l), \tag{27}$$

where A_l denotes the interarrival time of the SSMP customers in state l. The conditional expectation in the above is expressed as

$$\begin{aligned} E[e^{-s_1 Q_{n+1}} e^{-s_2 A_l} | S_n = l] &= \int_0^\infty \int_0^\infty e^{-s_1 t_1} e^{-s_2 t_2} P(Q_{n+1} = t_1, A_l = t_2 | S_n = l) dt_1 dt_2 \\ &= \int_0^\infty e^{-s_2 t_2} E[e^{-s_1 Q_{n+1}} | A_l = t_2, S_n = l] dA_l(t_2). \end{aligned} \tag{28}$$

By conditioning on X_n and X_{n+1} again the conditional expectation in (28) is given by

$$\begin{aligned} & E[e^{-s_1 Q_{n+1}} | A_l = t_2, S_n = l] \\ &= \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} E[e^{-s_1 Q_{n+1}} | X_n = i, S_n = l, X_{n+1} = j, A_l = t_2] P(X_n = i, X_{n+1} = j | S_n = l, A_l = t_2) \\ &= \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} (s_1^*)^{j+1} P(X_n = i | S_n = l) P_{i+1,j}(t_2) \\ &= s_1^* \sum_{i=0}^{\infty} P(X_n = i | S_n = l) \Gamma_{i+1}(s_1^*, t_2), \end{aligned} \tag{29}$$

where $s_1^* = \mu/(s_1 + \mu)$. Substituting this into (28) yields

$$\begin{aligned} & E[e^{-s_1 Q_{n+1}} e^{-s_2 A_l} | S_n = l] P(S_n = l) \\ &= s_1^* \sum_{i=0}^{\infty} P(i, l) \int_0^\infty e^{-s_2 t_2} \Gamma_{i+1}(s_1^*, t_2) dA_l(t_2) \\ &= \frac{s_1^*}{2\pi \mathrm{i}} \sum_{i=0}^{\infty} P(i, l) \int_{Br} \gamma_{i+1}(s_1^*, s) \alpha_l(s_2 - s) ds \\ &= \frac{1}{2\pi \mathrm{i}} \int_{Br} \frac{(s_1^*)^2 \Phi_l(s_1^*) - (1 - s_1^*) H_l(s)}{s - h(s_1^*)} \alpha_l(s_2 - s) ds. \end{aligned} \tag{30}$$

Finally by substituting (30) into (27), we obtain the joint LST of A_n and Q_{n+1} as

$$E[e^{-s_1 Q_{n+1}} e^{-s_2 A_n}] = \sum_{l=1}^{M} \frac{1}{2\pi i} \int_{Br} \frac{(s_1^*)^2 \Phi_l(s_1^*) - (1 - s_1^*) H_l(s)}{s - h(s_1^*)} \alpha_l(s_2 - s) ds. \tag{31}$$

Thus we can calculate $E[A_n Q_{n+1}]$, and then $E[D^2]$ by (26).

2.6 Numerical Examples

In this section we illustrate the results of analysis in Sections 2.1–2.5 numerically by assuming that the sojourn time in state l follows exponential distribution with rate α_l ($l = 1, 2$) for the case $M = 2$. In this case, we have $dA_l(t) = \alpha_l e^{-\alpha_l t}$, and then the complex integral (6) reduces to

$$\int_0^\infty \Gamma_{i+1}(z, t) dA_l(t) = \alpha_l \gamma_{i+1}(z, \alpha_l), \tag{32}$$

which is free from the Bromwich integral.

We need to solve the simultaneous equations (7) for $\Phi_1(z)$ and $\Phi_2(z)$. Adding them yields $\Phi(z)$ that contains two unknowns $H_1(\alpha_1)$ and $H_2(\alpha_2)$. The values of these unknowns may be determined by first applying the normalization condition $\Phi(1) = 1$ and then, since $\Phi(z)$ is analytic in $|z| \leq 1$, forcing the zeros of the numerator in the unit circle to coincide with the zeros of the denominator in the expression for $\Phi(z)$. The expression for $\Phi(z)$ is then given by

$$\Phi(z) = \frac{H_1 \alpha_1 [(-1+z)(\lambda z - \mu) - z(1 - \kappa z)\alpha_2] + H_2 \alpha_2 [(-1+z)(\lambda z - \mu) - z(1 - \kappa z)\alpha_1]}{\alpha_1 z[(1 - z + pz)(\lambda z - \mu) + \alpha_2 z(1 - \kappa z)] - (\lambda z - \mu)[(-1+z)(\lambda z - \mu) - \alpha_2 z(1 - z + qz)]}, \tag{33}$$

where H_1 and H_2 denote $H_1(\alpha_1)$ and $H_2(\alpha_2)$, respectively, p and q are the (1,2) and (2,1) elements of P, respectively, and $\kappa = 1 - p - q$.

Let $T(z)$ be the denominator of $\Phi(z)$ in (33) which is a polynomial of the third degree. Clearly, $T(0) > 0, T(1) < 0, T(\mu/\alpha) > 0$ under the condition $\lambda + \alpha < \mu$, where

$$\alpha := \frac{1}{\pi_1/\alpha_1 + \pi_2/\alpha_2} = \frac{\alpha_1 \alpha_2 (p + q)}{\alpha_1 p + \alpha_2 q}. \tag{34}$$

is the average arrival rate of SSMP customers. Hence $T(z)$ has a unique zero in the unit circle. Using this requirement and the normalization condition $\Phi(1) = 1$, we find H_1 and H_2 as

$$H_1 = \frac{[\mu - z_1(\lambda + \mu + \alpha_1) + z_1^2(\lambda + \kappa\alpha_1)]\{q\alpha_2(-\lambda + \mu) + \alpha_1[p(-\lambda + \mu) + \alpha_2(-1 + \kappa)]\}}{(1 - \kappa)(1 - z_1)(\lambda z_1 - \mu)\alpha_1(\alpha_1 - \alpha_2)}, \tag{35}$$

$$H_2 = \frac{[-\mu + z_1(\lambda + \mu + \alpha_2) - z_1^2(\lambda + \kappa\alpha_2)]\{q\alpha_2(-\lambda + \mu) + \alpha_1[p(-\lambda + \mu) + \alpha_2(-1 + \kappa)]\}}{(1 - \kappa)(1 - z_1)(\lambda z_1 - \mu)\alpha_2(\alpha_1 - \alpha_2)}, \tag{36}$$

where z_1 is the unique solution to the equation $T(z) = 0$ in the unit circle.

We can now evaluate the mean waiting times of SSMP and Poisson customers as well as other performance measures. The following set of parameters is assumed: $p = \frac{2}{25}$, $q = \frac{3}{25}$, $\alpha_2 = \frac{1}{2}\alpha_1$, and $\mu = 10.0$. We plot the values of performance measures by changing the arrival rate α_1 of SSMP, which means that α also changes. We show the results for several values of Poisson

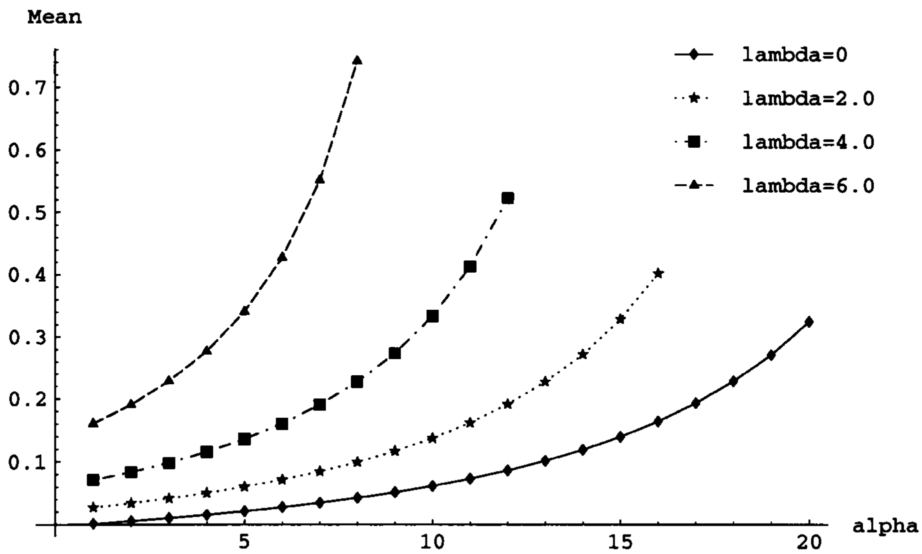

Figure 3: Mean waiting time for SSMP customers.

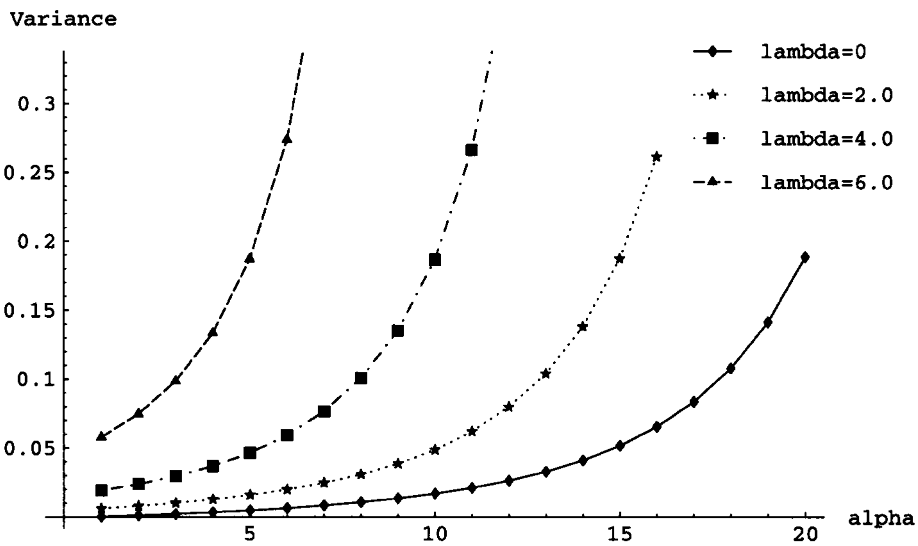

Figure 4: Variance of the waiting time for SSMP customers.

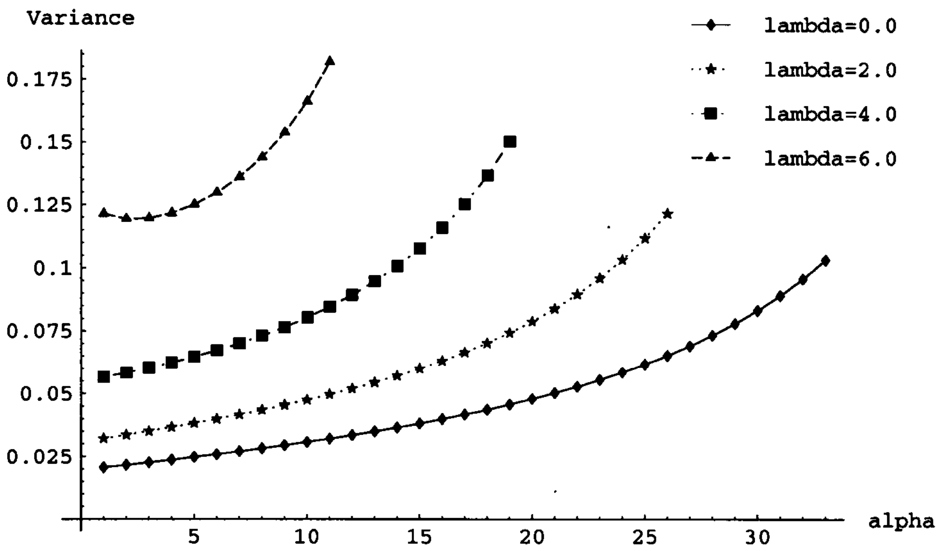

Figure 5: Variance of the jitter.

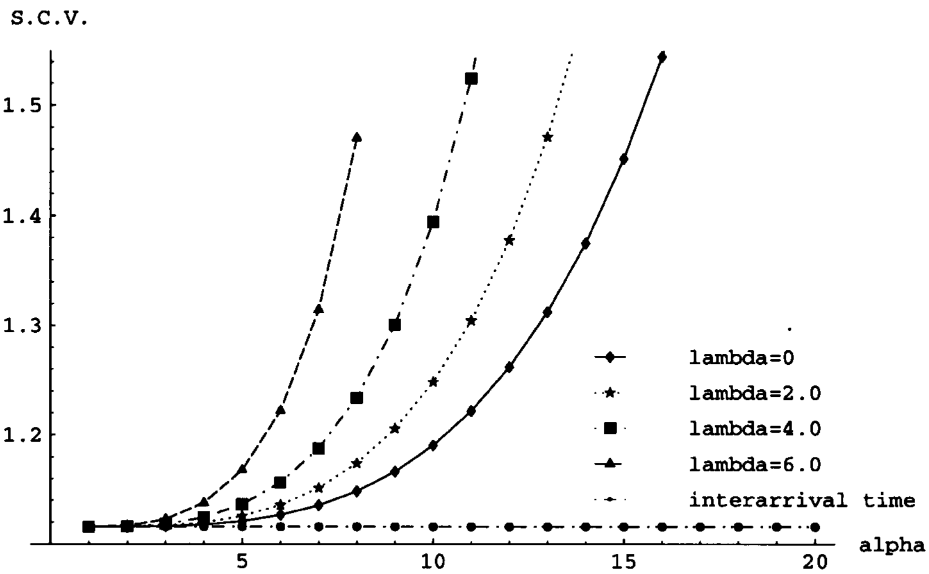

Figure 6: s.c.v. of the interdeparture and interarrival times.

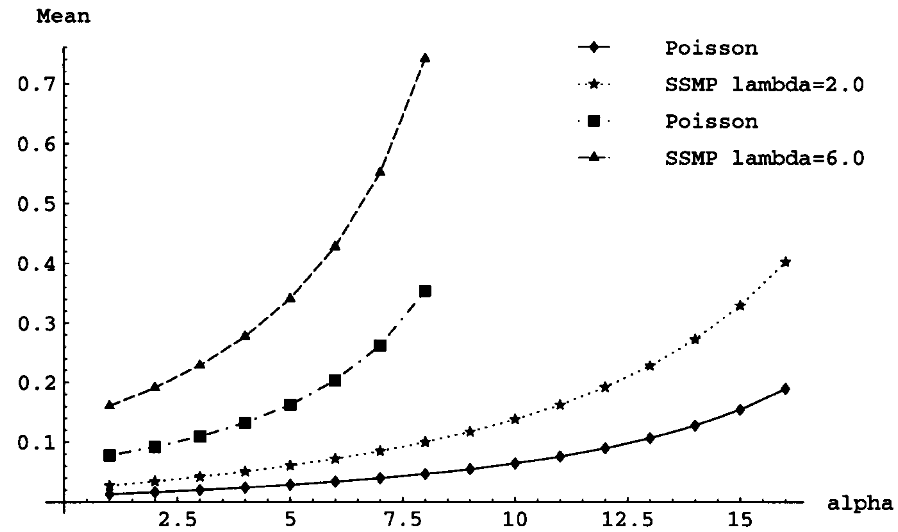

Figure 7: Mean waiting time for SSMP and Poisson customers.

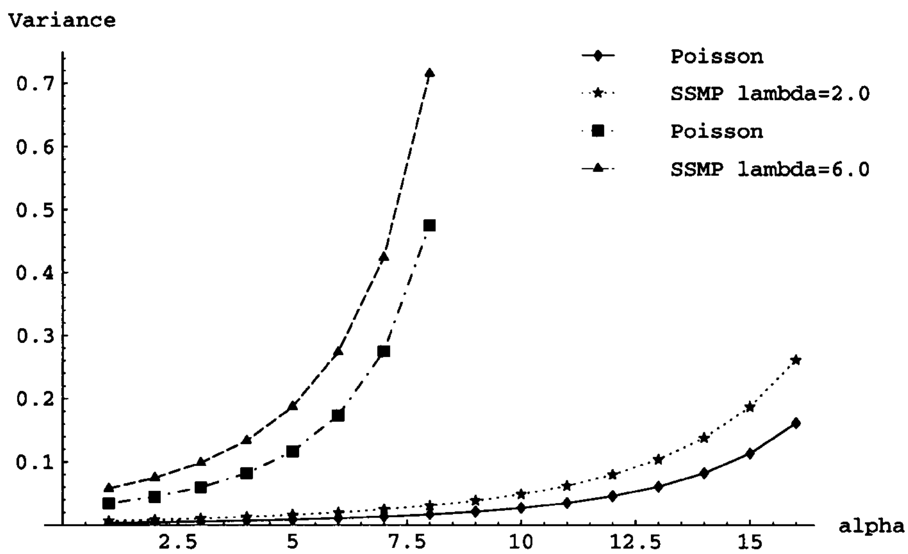

Figure 8: Variance of the waiting time for SSMP and Poisson customers.

arrival rate λ in the same figure in order to observe the influence by the interfering traffic. In Figures 3 and 4 the mean and variance of the waiting time of SSMP customers are shown respectively. Given the interfering traffic λ, the mean and variance increase as α gets big. We can also observe the influence of the interfering traffic on the waiting time of SSMP customers.

The mean of jitter must be 0 by definition as mentioned in Section 2.5, which has been confirmed by numerical calculation. Figure 5 shows the way the variance of jitter becomes big with the increase of α. We also plot the squared coefficient of variation (s.c.v.) of interdeparture time and of interarrival time of SSMP customers together in Figure 6. The s.c.v. of the interarrival time remains constant regardless of the value of α. The s.c.v. of the interdeparture time becomes big with the increase of α, and is always bigger than the s.c.v. of the interarrival time.

In Figures 7 and 8 we compare the mean and the variance of the waiting times for SSMP and Poisson customers for $\lambda = 2.0$ and $\lambda = 6.0$. It is observed that SSMP customers always receive worse service, i.e., bigger mean and variance. This is because the s.c.v. of the SSMP arrival process is bigger than that of the Poisson process which is unity. Note that the s.c.v. of the interarrival times of SSMP customers is bigger than 1 as shown in Figure 6. In Kuczura's study [7], it is reported that the arrival process having bigger s.c.v. receives worse service than that with smaller s.c.v., which agrees with our result.

3 Modeling of MPEG Video Traffic

In this section we present a queueing model for evaluating the waiting time of an arbitrary ATM cell generated from the frames of MPEG sequence in the presence of interfering traffic. An SSMP batch arrival process is assumed such that the SSMP has three states corresponding to the I, P, and B frames, and the batch accounts for a group of ATM cells from each frame. In Section 3.1 a brief description of MPEG coding scheme is given. An analysis of the $\mathrm{SSMP}^{[X]}+\mathrm{M/M/1}$ queueing system is shown in Section 3.2. In Section 3.3 we determine the state transition probabilities of the SSMP with three states as mentioned above. Assuming that the frame arrival process is Poisson we can obtain the formulas for evaluating the waiting time of an arbitrary ATM cell in the frame. Numerical examples using the statistics of real video films are presented in Section 3.4.

3.1 MPEG Video Coding Scheme

In the MPEG coding [4], a video traffic is compressed using the following three types of frames.

- I-frames are generated independently of P- or B-frames and inserted periodically.
- P-frames are encoded for the motion compensation with respect to the previous I- or P-frame.
- B-frames are similar to P-frames, except that the motion compensation can be with respect to the previous I- or P-frame, the next I- or P-frame, or an interpolation between them.

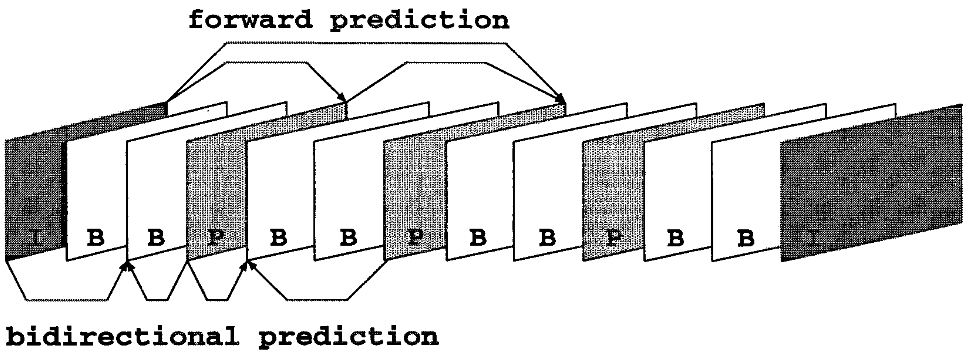

Figure 9: Group of pictures (GOP) of an MPEG stream [4].

These frames are arranged in a deterministic sequence "IBBPBBPBBPBB" as shown in Figure 9, which is called the Group of Pictures (GOP). The length of a GOP in Figure 9 is 12 frames. It is expected that this coding scheme leads to the statistical properties that are typical for MPEG video traffic stream. Table 1 contains some statistics of the frame size for the Jurassic Park (dino), the Soccer World Cup Final (soccer), and the Star Wars (star). These data are used in [17], and can be obtained from ftp://ftp-info3.informatik.uni-wuerzburg.de/pub/MPEG/.

The statistics in Table 1 for the number of ATM cells of each frame has been calculated by assuming that every frame is divided into a group of cells each with a payload of 48 bytes. It is observed that there is clear difference in the frame size among three types of frames I, P and B. Namely, the I-frames require much more bits than the P-frames. The B-frames have the lowest bits requirement. Thus the traffic stream generated by the MPEG coding is mainly characterized by two features, (i) deterministic frame pattern in GOP, and (ii) distinguishable frame size distributions for the three kinds of frames (I, P and B). In Section 3.3, we propose a traffic model containing these two features of MPEG coding.

	I frame			P frame			B frame		
video	mean	var	c.v.	mean	var	c.v.	mean	var	c.v.
dino	143.4	918.7	0.211	37.7	632.6	0.667	19.0	135.0	0.612
soccer	206.1	4321.6	0.319	123.7	3584.6	0.484	33.8	268.7	0.485
star	114.6	1355.6	0.321	26.4	618.1	0.942	12.1	123.4	0.918

Table 1: Statistics for the frame size in ATM cells.

3.2 Analysis of an SSMP$^{[X]}$+M/M/1 Queueing System

Let us consider the case where SSMP customers arrive in batch. Each arrival point corresponds to the arrival of a batch of customers, where the batch size (the number of customers in a batch) is an independent and identically distributed random variable. A random variable of batch size may be different according to the state of SSMP in which the batch arrives. We denote such an arrival process by SSMP$^{[X]}$. We study the waiting time of an arbitrary customer in an SSMP$^{[X]}$+M/M/1 queueing system.

Let $g_l(k)$ be the probability that the size of a batch that arrives in state l with duration A_l is k, where $k = 1, 2, \ldots$. Let $G_l(z)$ be the generating function of $g_l(k)$. The analysis is the same as that for the SSMP+M/M/1 system given in Section 2.2, except that we now consider the state immediately before the batch arrival.

By looking at the arrival time of an SSMP$^{[X]}$ batch, the one-step transition probability $r_{i,j}^{l,m}$ is given by

$$r_{i,j}^{l,m} = p_{l,m} \sum_{k=1}^{\infty} g_l(k) \int_0^{\infty} P_{i+k,j}(t) dA_m(t). \tag{37}$$

where $P_{i,j}(t)$ is given in (1). The probability $P(j,m) := P\{(X,S) = (j,m)\}$ satisfies the balance equations

$$P(j,m) = \sum_{i=0}^{\infty} \sum_{l=1}^{M} \sum_{k=1}^{\infty} p_{l,m} g_l(k) P(i,l) \int_0^{\infty} P_{i+k,j}(t) dA_m(t)$$

for $j = 0, 1, 2, \ldots$; $m = 1, 2, \ldots, M$. By transforming the above equations into the PGF with respect to $X = j$ as in Section 2.2, we obtain

$$\begin{aligned}
\Phi_m(z) &:= \sum_{j=0}^{\infty} P(j,m) z^j = \sum_{i=0}^{\infty} \sum_{l=1}^{M} p_{l,m} \sum_{k=1}^{\infty} g_l(k) P(i,l) \int_0^{\infty} \Gamma_{i+k}(z,t) dA_m(t) \\
&= \sum_{l=1}^{M} p_{l,m} \frac{1}{2\pi \mathrm{i}} \int_{Br} \sum_{i=0}^{\infty} P(i,l) \sum_{k=1}^{\infty} g_l(k) \gamma_{i+k}(z,s) \alpha_m(-s) ds \\
&= \sum_{l=1}^{M} p_{l,m} \frac{1}{2\pi \mathrm{i} z} \int_{Br} \left[\frac{z G_l(z) \Phi_l(z) - (1-z) H_l(s)}{s - h(z)} \right] \alpha_m(-s) ds,
\end{aligned} \tag{38}$$

where

$$H_l(s) := \frac{\eta(s) G_l(\eta(s)) \Phi_l(\eta(s))}{1 - \eta(s)}.$$

From the solution of this equation for $\Phi_m(z)$, we can obtain

$$\Phi(z) = \sum_{m=1}^{M} \Phi_m(z). \tag{39}$$

as the PGF for the number of customers that the first customer in a batch finds in the system.

Let us consider a randomly chosen *tagged* customer included in a batch that arrives during state l. Note that the number G_l^- of customers placed before the tagged customer within the batch is equivalent to the backward recurrence time in the (discrete-time) Markov renewal process, where the interval between two successive Markov renewal points corresponding to the batch size. Thus the PGF $G_l^-(z)$ of G_l^- is given by [19]

$$G_l^-(z) = \frac{1 - G_l(z)}{g_l(1-z)} \tag{40}$$

where g_l is the mean batch size. The LST $D_l(s)$ of the waiting time distribution for the tagged customer within his batch is then given by

$$D_l(s) = \sum_{k=0}^{\infty} [B(s)]^k P(G_l^- = k) = G_l^-[B(s)] = \frac{1 - G_l[B(s)]}{g_l[1 - B(s)]}, \tag{41}$$

where $B(s) = \mu/(s + \mu)$. Since the waiting time of the first customer in the batch and the waiting time of the tagged customer after the service to that batch has started are independent, the waiting time of the tagged customer is given by adding those waiting times. Thus its LST is given by $\Phi_l[B(s)]D_l(s)$. Hence we get the overall LST of the waiting time distribution for an arbitrary SSMP customer as

$$\Omega(s) = \frac{\sum_{l=1}^{M} \pi_l g_l \Phi_l[B(s)]D_l(s)}{\sum_{l=1}^{M} \pi_l g_l} = \frac{\sum_{l=1}^{M} \pi_l \Phi_l[B(s)](1 - G_l[B(s)])}{g(1 - B(s))}. \tag{42}$$

where $g := \sum_{l=1}^{M} \pi_l g_l$ is the overall mean batch size. The mean waiting time $E[W]$ and the second moment $E[W^2]$ for an arbitrary SSMP customer are then given by

$$E[W] = \frac{1}{g\mu} \sum_{l=1}^{M} \pi_l \left(g_l E_l[X] + \frac{g_l^{(2)}}{2} \right), \tag{43}$$

$$E[W^2] = \frac{1}{g\mu^2} \sum_{l=1}^{M} \pi_l \left((E_l[X] + E_l[X^2])g_l + (1 + E_l[X])g_l^{(2)} + \frac{g_l^{(3)}}{3} \right), \tag{44}$$

where $g_l^{(2)} = \frac{d^2}{dz^2} G_l(z)\big|_{z=1}, g_l^{(3)} = \frac{d^3}{dz^3} G_l(z)\big|_{z=1}$, and $E_l[X]$ and $E_l[X^2]$ are calculated from $G_l(z)$.

3.3 Traffic Model for MPEG Video Sequences

We consider a batch arrival system with three states treated in Section 3.2. These states correspond to the I, P and B-frames. We determine the values for the elements of state transition probability matrix P so as to match the frame appearance frequency in a GOP. It is evident from Figure 9 that I-frames are always followed by B-frames and that P-frames by B-frames. Thus we set the transition probability from I to B to 1 and those to all others to 0. The transition probabilities from P are the same as those from I. We also observe that B-frames are followed by I, P and B-frames. Taking the frequency of transitions from B-frames into account, we determine the transition probabilities from B as follows: $p_{\mathrm{B,I}} = 1/8, p_{\mathrm{B,B}} = 1/2$, and $p_{\mathrm{B,P}} = 3/8$. Hence we have

$$P = \begin{array}{c c} & \begin{array}{ccc} \mathrm{I} & \mathrm{B} & \mathrm{P} \end{array} \\ \begin{array}{c} \mathrm{I} \\ \mathrm{B} \\ \mathrm{P} \end{array} & \left[\begin{array}{ccc} 0 & 1 & 0 \\ \frac{1}{8} & \frac{1}{2} & \frac{3}{8} \\ 0 & 1 & 0 \end{array} \right] \end{array} . \tag{45}$$

For the sake of simplicity in the expressions, we assume that the arrival process of the frames is Poisson with rate α, as a special case of SSMP. Let $G_I(z)$ and g_I be the PGF and the mean size of the cells generated from I-frames. Let $G_P(z)$, $G_B(z)$, g_P, and g_B be defined similarly. Solving equations (38) we obtain $\Phi_m(z)$ $(m = \mathrm{I}, \mathrm{B}, \mathrm{P})$. Summing these up as in (39) yields the PGF for the number of frames present in the system at arrival times as

$$\Phi(z) = \frac{(1-z)\{C[\alpha z G_B(z) - 2q(z)] - 3\alpha^2 z H[G_I(z) - 4G_B(z) + 3G_P(z)]\}}{24q^2(z) + 3\alpha z G_B(z)\{4q(z) - \alpha z[G_I(z) + 3G_P(z)]\}}, \tag{46}$$

where

$$q(z) = \lambda z^2 - (\alpha + \lambda + \mu)z + \mu,$$
$$C = 12\lambda - 12\mu + \alpha g_I + 8\alpha g_B + 3\alpha g_P.$$

The unknown H in the above equation can be found as follows. Consider the following equation from the denominator of (46):

$$8q^2(z) + \alpha z G_B(z)\{4q(z) - \alpha z[G_I(z) + 3G_P(z)]\} = 0. \tag{47}$$

It is shown in the Appendix that there are two solutions ($|z| \leq 1$) to (47) under the condition $\lambda/\mu + \alpha g/\mu < 1$, one of which is $z = 1$. Let z_1 be the other one. By forcing the zero in the numerator of (46) in the unit circle coincide with z_1, the unknown H is found to be

$$H = \frac{C[\alpha z_1 G_B(z_1) - 2q(z_1)]}{3\alpha^2 z_1[G_I(z_1) - 4G_B(z_1) + 3G_P(z_1)]}. \tag{48}$$

This completes the determination of parameters in the model.

3.4 Numerical Examples

Let us evaluate the waiting time of an arbitrary cell in the model proposed in the preceding section. We need to assume some distribution function for the number of cells in each frame so that we can calculate the value of z_1 numerically as the solution to (47). Let us assume that the distribution for the number of cells in each frame is negative binomial whose parameters are determined from the mean and variance of the actual data. We also assume that cells are transmitted on a 10Mbps (2,350 cells/sec) channel.

Figures 10 and 11 show the mean and the variance of the waiting time of an arbitrary ATM cell in the MPEG frames for the Jurassic Park in the presence of interfering traffic. It is observed that at low arrival rate α (frames/sec) both the mean and variance are flat, but at high load they increase rapidly with α. We can also observe the degree of influence by an interfering traffic, where its rate λ is given in the unit of cells/sec. Figures 12 and 13 are for the World Cup Final, and Figures 14 and 15 for Star Wars.

In the MPEG coding each encoded frame is transmitted with a rate of about 25 to 30 frames/sec. We can estimate from Figures 10 and 14 that the mean cell waiting time is about 20ms regardless of the value of λ for the Jurassic Park and the Star Wars. From the result for the soccer in Figure 12 it is observed that the mean cell waiting time is moderate when there is no interfering traffic ($\lambda = 0$ cells/sec) and the waiting time becomes extremely large when there are much interfering traffic ($\lambda = 600$ or 900 cells/sec). A 10Mbps channel may not be enough in this case.

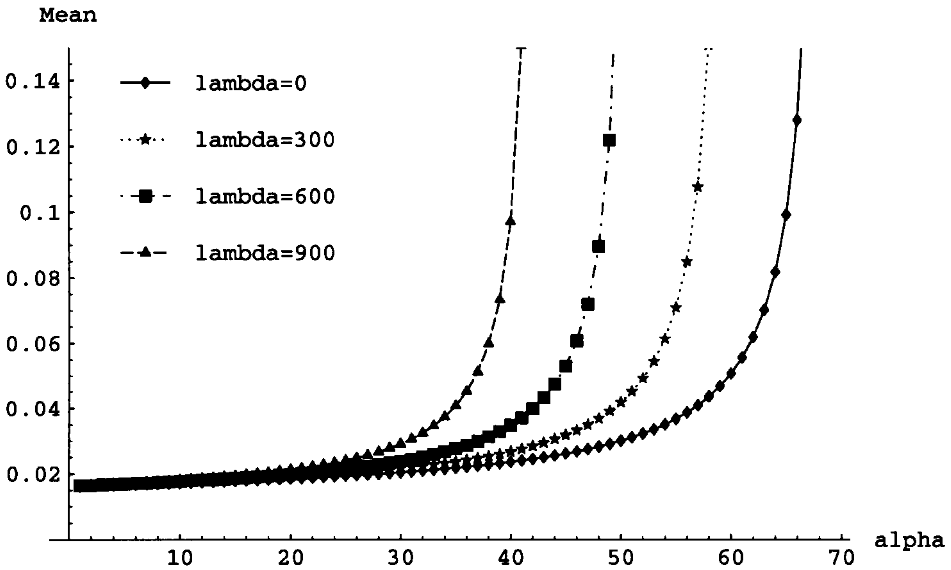

Figure 10: Mean waiting time for an arbitrary cell [sec] (dino).

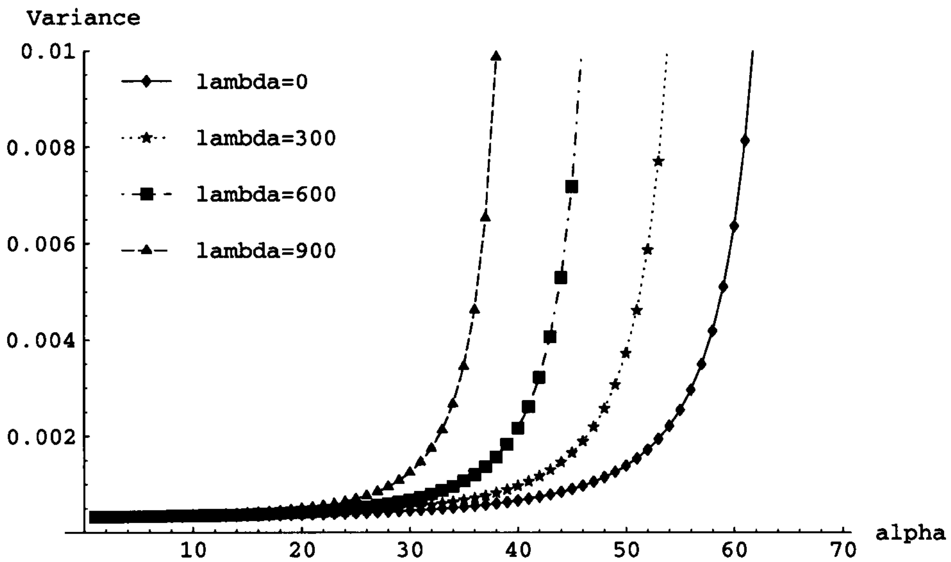

Figure 11: Variance of the waiting time for an arbitrary cell (dino).

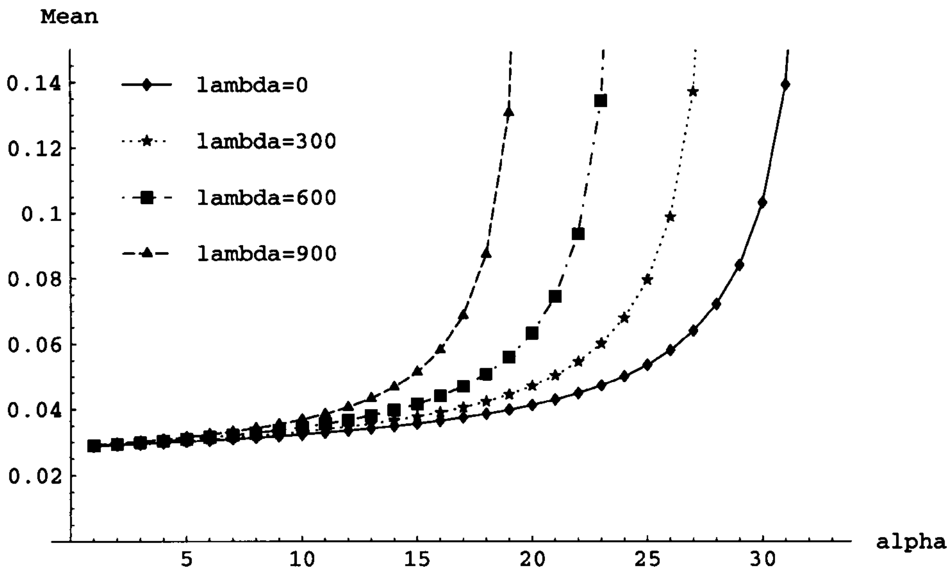

Figure 12: Mean waiting time for an arbitrary cell [sec] (soccer).

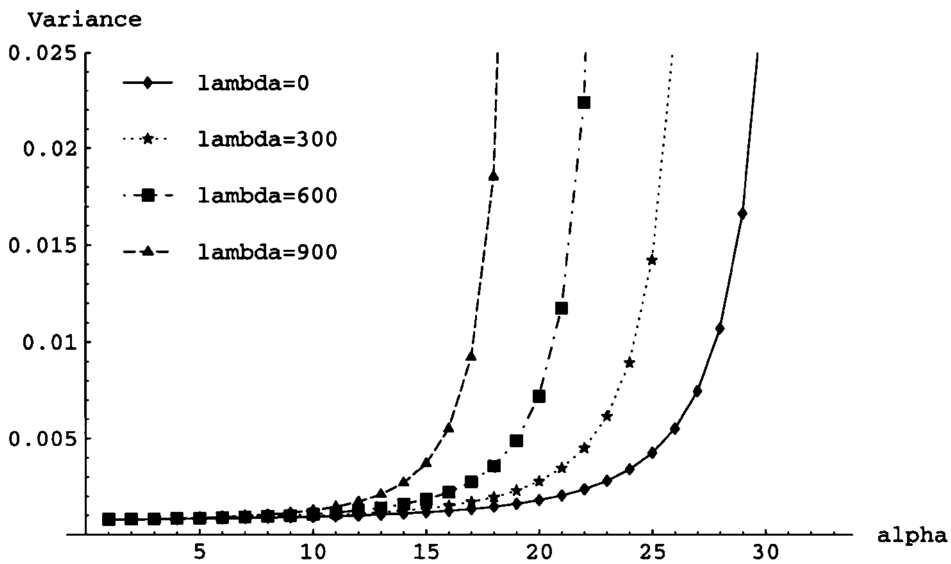

Figure 13: Variance of the waiting time for an arbitrary cell (soccer).

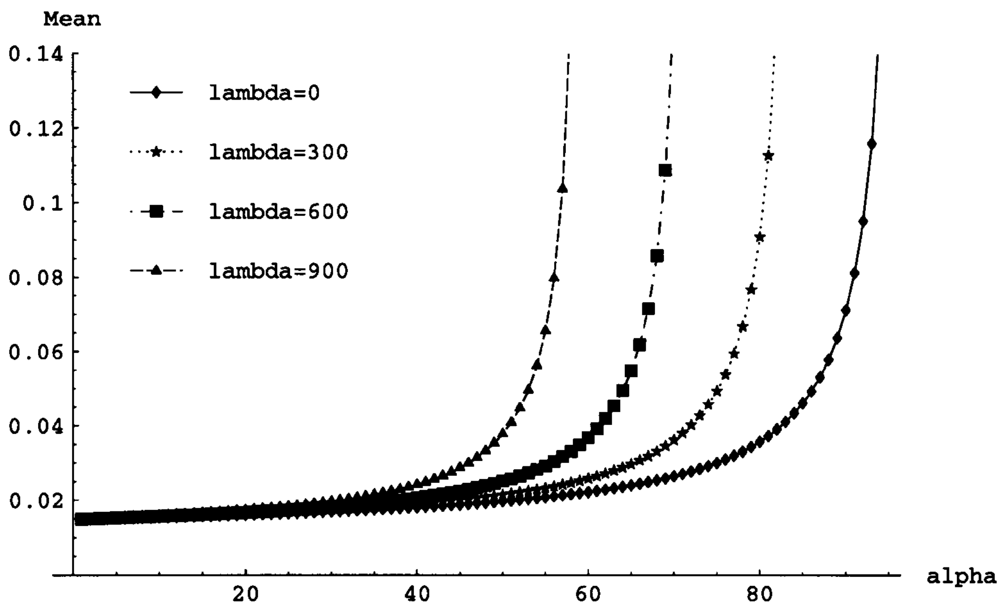

Figure 14: Mean waiting time for an arbitrary cell [sec] (starwars).

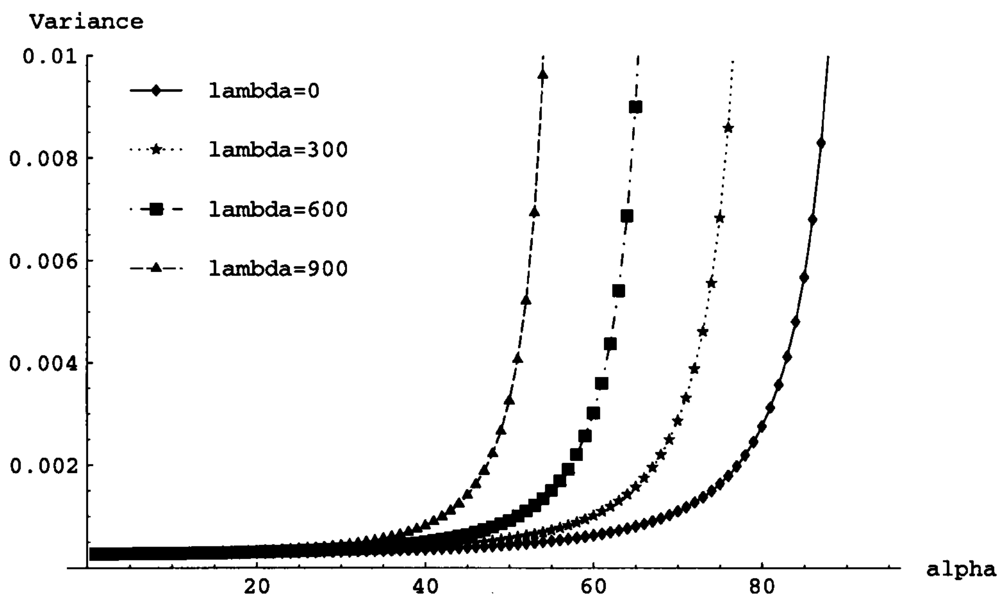

Figure 15: Variance of the waiting time for an arbitrary cell (starwars).

4 Summary

In this paper we have studied a queueing system having a mixture of SSMP and Poisson arrivals as an input process, where the Poisson arrival is regarded as an interfering traffic. It is shown by numerical examples that the SSMP arrivals receive worse service than Poisson arrivals, i.e. the mean waiting time of SSMP customers is longer than Poisson customers.

We have also proposed a model of MPEG frame arrivals by using an SSMP batch arrival process. This model includes two features of the MPEG coding scheme: (i) deterministic frame pattern in GOP, and (ii) distinct frame size distributions for the I, P and B frames. In the numerical examples, the waiting time of each cell generated from frame is evaluated based on this model. It is observed that both the mean and variance are flat at low arrival rate, but they increase rapidly with the increase in the frame arrival rate. It is also found that the waiting time is very different among the three video data. Especially for the soccer data, though the waiting time is moderate when there is no interfering traffic, it becomes extremely large when the interfering traffic is high.

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Appendix Zeros of the Denominator of (46)

We show that the denominator of equation (46)

$$\begin{aligned} T(z) &:= 8q^2(z) + \alpha z G_B(z)\{4q(z) - \alpha z[G_I(z) + 3G_P(z)]\} \\ &= 4q(z)[2q(z) + \alpha z G_B(z)] - \alpha^2 z^2 G_B(z)[G_I(z) + 3G_P(z)] \end{aligned} \tag{A.1}$$

has exactly 2 zeros on the unit disk of the complex plane, one of which is unity, when the inequality $1 - (\lambda/\mu + \alpha g/\mu) > 0$ holds. The proof is based on Rouché's theorem: *If $d(z)$ and $h(z)$ are analytic functions of z inside and on a closed contour C, and $|h(z)| < |d(z)|$ on C, then $d(z)$ and $d(z) - h(z)$ have the same number of zeros inside C.* We prove the above claim in a way similar to that in [6].

Let

$$d(z) := 4q(z)[2q(z) + \alpha z G_B(z)], \tag{A.2}$$

$$h(z) := \alpha^2 z^2 G_B(z)[G_I(z) + 3G_P(z)]. \tag{A.3}$$

Then $T(z) = d(z) - h(z)$. The functions $d(z)$ and $h(z)$ are analytic in a part of the complex plane. Since $q(z) = \lambda z^2 - (\alpha + \lambda + \mu)z + \mu$, we have

$$|q(z)| \geq \alpha + \lambda + \mu - \lambda - \mu = \alpha \qquad \text{on } |z| = 1.$$

We set $z = e^{i\theta}$ $(0 \leq \theta < 2\pi)$ on the peripheral of the unit circle. Then

$$|d(e^{i\theta})| \geq 4\alpha(2\alpha - \alpha) = 4\alpha^2, \tag{A.4}$$

$$|h(e^{i\theta})| < \alpha^2(1+3) = 4\alpha^2 \qquad (\theta \neq 0), \tag{A.5}$$

because $G_l(z)$'s are PGFs ($|G_l(e^{i\theta})| < 1$ for $\theta \neq 0$) for $l =$ I, B, P. Hence, $|h(z)| < |d(z)|$ for all $z \neq 1$ on the unit circle.

Let us choose a contour C so as to include $z = 1$ as an internal point, which is obviously a zero of $T(z)$. In particular, we choose the contour C as

$$C \equiv \left\{ z = e^{i\theta}; 0 < \theta < 2\pi \right\} \cup \lim_{\varepsilon \to 0} C_\varepsilon, \tag{A.6}$$

where

$$C_\varepsilon \equiv \left\{ z = 1 + \varepsilon e^{i\beta}; -\frac{\pi}{2} < \beta < \frac{\pi}{2} \right\} \tag{A.7}$$

is a semicircle centered at $z = 1$ with radius $\varepsilon > 0$. Let $z = 1 + \varepsilon e^{i\beta}$ for $z \in C_\varepsilon$. Then $q(z) = \lambda(1+\varepsilon e^{i\beta})^2 - (\alpha+\lambda+\mu)(1+\varepsilon e^{i\beta}) + \mu = -\alpha + (\lambda - \alpha - \mu)\varepsilon e^{i\beta} + o(\varepsilon)$, so it follows that

$$\begin{aligned} |d(z)|^2 &= \Big| 4(-\alpha + (\lambda - \alpha - \mu)\varepsilon e^{i\beta} + o(\varepsilon)) \big(-2\alpha + 2(\lambda - \alpha - \mu)\varepsilon e^{i\beta} + o(\varepsilon) \\ &\quad + \alpha(1 + \varepsilon e^{i\beta})(1 + G_B'(1)\varepsilon e^{i\beta} + o(\varepsilon)) \big) \Big|^2 \\ &= 16|\alpha^2 + \alpha(2\alpha - g_B\alpha + 3\mu - 3\lambda)\varepsilon e^{i\beta} + o(\varepsilon)|^2 \\ &= 16\alpha^4 + 32\alpha^3(2\alpha - g_B\alpha + 3\mu - 3\lambda)\varepsilon \cos\beta + o(\varepsilon). \end{aligned} \tag{A.8}$$

We also have

$$\begin{aligned} |h(z)|^2 &= |\alpha^2(1+\varepsilon e^{i\beta})^2(1 + g_B\varepsilon e^{i\beta} + o(\varepsilon))(1 + g_I\varepsilon e^{i\beta} + 3(1 + g_P\varepsilon e^{i\beta}) + o(\varepsilon))|^2 \\ &= \alpha^4|4 + (g_I + 4g_B + 3g_P + 8)\varepsilon e^{i\beta} + o(\varepsilon)|^2 \\ &= 16\alpha^4 + 8\alpha^4(g_I + 4g_B + 3g_P + 8)\varepsilon\cos\beta + o(\varepsilon). \end{aligned} \tag{A.9}$$

Since $1 - (\lambda/\mu + \alpha g/\mu) > 0$, we have $|d(z)|^2 > |h(z)|^2$ (therefore $|d(z)| > |h(z)|$) on C_ε for a sufficiently small value of ε, and hence also on the entire contour C. Thus the functions $d(z), h(z)$ and the contour C satisfy the condition of Rouché's theorem. It follows that $d(z)$ and $d(z) - h(z) = T(z)$ have the same number of zeros inside C.

We consider the number of zeros of $d(z)$ inside C. There is only one zero inside C among the roots of $q(z) = 0$, which is $z = \left[\alpha + \lambda + \mu - \sqrt{(\alpha+\lambda+\mu)^2 - 4\lambda\mu}\,\right]/2\lambda$. We apply Rouché's theorem to $2q(z) + \alpha z G_B(z)$ again, and find that $2q(z) + \alpha z G_B(z)$ has one zero inside C. Thus $d(z)$ has two zeros inside C. This implies that $T(z)$ also has two zeros by Rouché's theorem, one of which is obviously $z = 1$.